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THERMAL RADIATION FROM A CYLINDRICAL ENCLOSURE WITH SPECIFIED WALL HEAT FLUX

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ABSTRACT

Thermal radiation is considered from the inside surface of a heated cylindrical enclosure. A specified heat input is imposed at the enclosure wall and the surface temperature distribution is determined. The analysis considers both black and diffuse gray body radiation and obtains a simple relation between the two. The energy equation, which is a linear integral equation, is solved by three methods: (a) use of a separable kernel, (b) numerical integration, and (c) variational methods.

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INTRODUCTION

Introducing radiation into heat transfer problems results in changing the energy equation from a differential equation to an integro-differential equation. For the case where heat transfer is only by radiation, which we consider here, the energy equation simplifies to a linear integral equation in the variable U or, for black bodies, T^4. Although we consider only a special case in pure radiation, the methods presented here could be applied to other similar radiation problems.

Specifically we consider thermal radiation from the internal surface of a cylindrical enclosure open at both ends as shown in Fig. 1. The internal surface is heated by a specified heat flux which is a function of axial position, and the wall temperature is found from the analysis. The radiation from the surface may either be black or diffuse gray body. Heat conduction within the enclosure walls is assumed negligible compared with the radiation fluxes, and the enclosure is assumed to be in a vacuum so that heat convection does not enter the problem. Each end of the cylinder sees an outside environment which may each be at a different specified temperature.

With the above assumptions the energy equation governing the radiation process is found to be a linear integral equation, and the linearity permits building more

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general solutions by superposition of elementary situations. The necessary elementary solutions are found by three different methods, (a) use of an approximate separable kernel, (b) numerical integration, and (c) a variational method.

In the analysis we first consider the black body case and then extend the results to include the more general situation of diffuse gray body radiation.

NOMENCLATURE

A,B,C,D,E	coefficients determined by the variational method
d	length given on Fig. 2
dA_1, dA_2	elementary areas on cylindrical surface (see Fig. 2)
F	configuration factor for a circular element radiating to a perpendicular element
J	variational principle for Φ; J_q, variational principle for Φ_q
K	area factor kernel in integral equation
l	length of cylindrical enclosure
L	dimensionless length, $l/2R$
M,N,P	functions of L defined in Eq. (13)
q	heat added per unit area at enclosure wall
R	radius of cylindrical enclosure
r	radial coordinate measured from cylinder axis
T	absolute temperature; T_w, absolute wall temperature; T_1, T_2, temperatures of environments seen at ends of enclosure
U	total radiation leaving surface per unit area
x	dimensionless axial coordinate, $x'/2R$
x'	axial length coordinate measured from one end of enclosure
z	length defined as $\lvert x - \eta \rvert$
α	exponent in approximate separable kernel
β	exponent in approximation for F
ϵ	emissivity

η' dummy integration variable; η, dimensionless variable $\eta'/2R$

$\theta, \theta_1, \theta_2$ angles given on Fig. 2

ρ reflectivity

σ Stefan-Boltzmann constant

Φ wall emissive power for $q = 0$, and one end of enclosure exposed to unit emissive power

Φ_q wall emissive power for specified q at the walls; Φ_q^*, defined as Φ_q/q

φ emissive power (σT^4); φ_w, wall emissive power; φ_1, φ_2, emissive powers of environments seen at ends of enclosure

ENERGY BALANCE (BLACK BODY)

Consider the heat quantities entering and leaving an element dA_1 (see Fig. 2) located at position x on the internal surface of the cylindrical enclosure. The heat radiated from the element per unit area is given by the emissive power (we are here considering only black body radiation),

$$\sigma T_w^4(x) \tag{1}$$

The heat input to the surface element is composed of four quantities. The first is the imposed heat flux at the wall, which on a unit area basis is simply $q(x)$. The second and third are the heat inputs q_1 and q_2 from the constant environment temperatures T_1 and T_2 at the ends of the enclosure. These environments can be replaced by fictious black body surfaces at the ends of the cylinder. The configuration factor which then applies is for a circular area radiating to an element in a plane perpendicular to the circle and this has been given in Ref. (1). Hence, we can immediately write

$$q_1 = \sigma T_1^4 \, F(x) \tag{2a}$$

$$q_2 = \sigma T_2^4 \, F(L - x) \tag{2b}$$

where

$$F(x) = \left[\frac{x^2 + (1/2)}{(x^2 + 1)^{1/2}} - x \right]$$

Note that throughout the analysis, all length quantities have been made dimensionless by dividing by the ~~tube~~ cylinder diameter. The fourth heat input is the radiation to dA_1

from all other elements such as dA_2 on the internal surface of the cylinder. This is obtained by integrating, over the enclosure, the product of the emissive power of dA_2 and the configuration factor between dA_2 and dA_1. The details of the mathematical manipulation are given in the appendix and result in the following expression

$$\int_0^x \sigma T_w^4(\eta) K(x-\eta)\, d\eta + \int_x^L \sigma T_w^4(\eta) K(\eta-x)\, d\eta$$

where

$$K(z) = 1 - \frac{z^3 + \frac{3}{2} z}{(z^2 + 1)^{3/2}} \qquad z \geq 0 \tag{3}$$

If we then form the heat balance and let the emissive power, φ, replace σT^4, the following linear integral equation is obtained,

$$\varphi_w(x) = q(x) + \varphi_1 F(x) + \varphi_2 F(L - x) + \int_0^x \varphi_w(\eta) K(x - \eta)\, d\eta + \int_x^L \varphi_w(\eta) K(\eta - x)\, d\eta \tag{4}$$

We now proceed to solve this equation for $\varphi_w(x)$ when $q(x)$ is specified.

SUPERPOSITION OF SOLUTIONS (BLACK BODY)

Since the energy equation is linear in φ, a complete solution can be formed by superposing three elementary solutions as illustrated in Fig. 3. In the first problem the surroundings are all at φ_2 and there is no imposed heat input at the enclosure surface. The surface must then come to equilibrium at the same temperature as the surroundings, and we have the trivial result that φ at the wall is equal to φ_2. In the second problem the surroundings are at absolute zero temperature and there is a heat input $q(x)$ at the wall. The solution for this situation will be designated by $\Phi_q(x)$. For the third problem the environment at one end of the enclosure is at a constant emissive power, $\varphi_1 - \varphi_2$, and $q(x)$ is zero. For the case when $\varphi_1 - \varphi_2$ has a value of unity the solution is termed $\Phi(x)$. By superposition the general solution to Eq. (4) is then

$$\varphi_w(x) = \varphi_2 + \Phi_q(x) + (\varphi_1 - \varphi_2)\Phi(x) \tag{5}$$

We shall first consider the solution for $\Phi_q(x)$ and then obtain $\Phi(x)$.

SOLUTION FOR SPECIFIED HEAT INPUT WITH SURROUNDINGS AT ZERO (BLACK BODY)

In this section the solution for $\Phi_q(x)$ will be found. We shall limit the discussion to the case where $q(x)$ is a constant although the methods used can be applied for the more general case of variable heat input with length. With $\varphi_1 = \varphi_2 = 0$, and letting $\Phi_q^*(x) = \Phi_q(x)/q$, Eq. (4) reduces to the following relation for $\Phi_q^*(x)$

$$\Phi_q^*(x) = 1 + \int_0^x \Phi_q^*(\eta)K(x - \eta)\, d\eta + \int_x^L \Phi_q^*(\eta)K(\eta - x)\, d\eta \tag{6}$$

Eq. (6) will now be solved using three different methods.

Approximate separable kernel. - The kernel $K(z)$ in Eq. (6) has been plotted on Fig. 4 and it is seen that it has the general shape of an exponential function. We then try to approximate $K(z)$ by the more convenient function $e^{-\alpha z}$. To choose the constant α, consider an infinite isothermal cylindrical enclosure. Then by imposing the condition that there be a heat balance on any element of the internal surface it is seen that

$$2\int_0^\infty K(z)\, dz = 1 \tag{7}$$

This condition is then applied to the approximate kernel so that

$$\int_0^\infty e^{-\alpha z}\, dz = 1/2 \tag{7a}$$

or $\alpha = 2$. The kernel e^{-2z} is given on Fig. 4 and it is seen that the agreement with the exact kernel is quite good. The approximation is then substituted into Eq. (6) with the result

$$\Phi_q^*(x) = 1 + \frac{1}{e^{2x}}\int_0^x \Phi_q^*(\eta)e^{2\eta}\, d\eta + e^{2x}\int_x^L \Phi_q^*(\eta)\,\frac{1}{e^{2\eta}}\, d\eta \tag{8}$$

The advantage of this method is now evident since by means of the separable kernel the quantities involving x have been taken out from under the integral signs, and the integral equation can now be solved quite easily. By differentiating the equation twice, the integrals can be removed and a simple differential equation is obtained

$$\frac{d^2\Phi_q^*}{dx^2} = -4 \tag{8a}$$

This has the general solution

$$\Phi_q^* = -2x^2 + C_1 x + C_2 \tag{9}$$

From symmetry we know that at $x = L/2$, $d\Phi_q^*/dx = 0$, and hence $C_1 = 2L$. To determine C_2 we evaluate the integral Eq. (8) at $x = 0$ and $x = L$ and utilize the fact that $T_w(0) = T_w(L)$. This yields the boundary condition,

$$\int_0^L \Phi_q^*(\eta)\, \frac{1}{e^{2\eta}}\, d\eta = \frac{1}{e^{2L}} \int_0^L \Phi_q^*(\eta)\, e^{2\eta}\, d\eta$$

Inserting $\Phi_q^* = -2x^2 + 2Lx + C_2$ into this relation gives $C_2 = L + 1$, so that the final result for Φ_q^* is the parabola

$$\Phi_q^* = L + 1 + 2(xL - x^2) \tag{9a}$$

The next step in the analysis was to test this simple expression by performing a direct numerical integration of Eq. (6) for a few sample cases.

Numerical integration. - The integrals in Eq. (6) were placed in finite difference form and numerical computations were carried out for enclosures with length-diameter ratios of 1 and 4. In the numerical method the enclosure length was divided into equal increments and the wall temperature at each position was expressed in terms of the temperatures at the other positions. This resulted in a system of simultaneous equations which were solved on an IBM 704 digital computer. The increment sizes were cut in half until convergence was obtained. Some of the final results are given in the table, and it is seen that although the temperatures obtained numerically deviate by several percent from the parabola obtained in the previous

section, the curves are still very close to being parabolic in shape. It appears then, that a parabola provides a good approximation to the solution of the integral equation. With this in mind we then turn to variational methods which provide a means for determining the parabola which best approximates the exact solution for an enclosure of any length.

Variational method. - As shown in Ref. 2, page 205, an integral equation of the Fredholm type, such as Eq. (6), can be solved by obtaining an extremum of the associated variational principle,

$$J_q = \int_0^L \int_0^L K(|\eta - x|)\, \Phi_q^*(\eta)\Phi_q^*(x)\, dx\, d\eta - \int_0^L \left[\Phi_q^*(x)\right]^2 dx + 2\int_0^L \Phi_q^*(x)\, dx \tag{10}$$

Since $\Phi_q^*(x)$ may be simply approximated by a parabola (a higher order polynomial could be used if necessary) we let

$$\Phi_q^*(x) = A + Bx + Cx^2 \tag{11}$$

Actually from symmetry about $x = L/2$ we know that $B = -CL$, but we shall use Eq. (11) in its present form since this will enable part of the work to be utilized again in a later section. Eq. (11) is then substituted into Eq. (10) and the integration is carried out. The algebra involved in the double integral is considerable and very lengthy expressions are obtained which will not be given here. The resulting expression gives J_q in terms of the three coefficients A,B, and C. To obtain an extremum we follow the Ritz procedure (see for example, Ref. 3, p. 187) and differentiate J_q separately with respect to each of the coefficients and set each of the three resulting equations equal to zero. This yields,

$$\frac{\partial J_q}{\partial A} = 0 \longrightarrow AL(-L + \sqrt{L^2 + 1}) + \frac{B}{2} L^2(-L + \sqrt{L^2 + 1}) +$$

$$\frac{C}{3}\left[-L^4 - L + \left(L^3 + \frac{1}{4}L\right)\sqrt{L^2 + 1} + \frac{3}{4} \ln\left(L + \sqrt{L^2 + 1}\right)\right] - L = 0 \tag{12a}$$

$$\frac{\partial J_q}{\partial B} = 0 \longrightarrow AL(-L + \sqrt{L^2+1}) + \frac{B}{2}\left[-L^3 + \frac{1}{2}\sqrt{L^2+1} - \frac{1}{2L}\ln\left(L + \sqrt{L^2+1}\right) + L^2\sqrt{L^2+1}\right] +$$

$$\frac{C}{3} L \left[-L^3 - 1 + (L^2 + 1)\sqrt{L^2 + 1}\right] - L = 0 \qquad (12b)$$

$$\frac{\partial J_q}{\partial C} = 0 \longrightarrow A\left[2L^4 + 2L - \frac{3}{2}\ln\left(L + \sqrt{L^2 + 1}\right) - \left(2L^3 + \frac{1}{2}L\right)\sqrt{L^2 + 1}\right] +$$

$$BL^2\left[(L^3 + 1) - (L^2 + 1)\sqrt{L^2 + 1}\right] +$$

$$C\left[\frac{2}{3}L^6 + \frac{4}{3}L^3 - \left(\frac{2}{3}L^5 + \frac{7}{6}L^3 + \frac{1}{4}L\right)\sqrt{L^2 + 1} +\right.$$

$$\left.\frac{1}{4}\ln\left(L + \sqrt{L^2 + 1}\right)\right] + 2L^3 = 0 \qquad (12c)$$

If Eq. (12a) is subtracted from (12b) the result immediately gives the relation $B = -CL$, which was mentioned before as necessary for the wall temperature distribution to be symmetric about $x = L/2$. Eqs. (12) are then solved simultaneously for A and C which gives the final form,

$$\Phi_q^* = \frac{M(L)}{P(L)} + \frac{N(L)}{P(L)}(xL - x^2) \qquad (13)$$

where

$$M(L) = \frac{2}{3}L^4 + \frac{1}{2}L^2\sqrt{L^2 + 1} - \left(L^3 + \frac{L}{2}\right)\ln\left(L + \sqrt{L^2 + 1}\right)$$

$$N(L) = -4L^2 + 3L\ln\left(L + \sqrt{L^2 + 1}\right) + L^2\sqrt{L^2 + 1}$$

$$P(L) = \frac{1}{6}L^6 + \frac{7}{12}L^4 - \frac{11}{12}L^2 +$$

$$\left[-\frac{1}{6}L^5 - \frac{1}{2}L^3 + \frac{2}{3}L^2\right]\sqrt{L^2 + 1} + \left[\frac{1}{2}L^4 + \frac{1}{2}L^2 + 2L\right]$$

$$\ln\left(L + \sqrt{L^2 + 1}\right) - \left(\frac{1}{2}L^3 + L\right)\sqrt{L^2 + 1}\ln$$

$$\left(L + \sqrt{L^2 + 1}\right) - \frac{3}{4}\left[\ln\left(L + \sqrt{L^2 + 1}\right)\right]^2$$

The distributions calculated from Eq. (13) are compared with the previous methods in the table, and good agreement is obtained. The coefficients $M(L)/P(L)$ and $N(L)/P(L)$ are plotted for L from 0 to 10 on Fig. 5a, and are compared with the

coefficients $L + 1$ and 2 from Eq. (9a). The results of the simple method using the separable kernel provide a good approximation.

SOLUTION FOR ZERO HEAT INPUT AND ONE END RESERVOIR HEATED (BLACK BODY)

Now the solution for $\Phi(x)$ is considered, which is for an enclosure with zero heat input at the walls, and with one end exposed to a reservoir at zero temperature and the other to an emissive power $\varphi = 1$. In this case $q(x) = 0$, $\varphi_2 = 0$, and $\varphi_1 = 1$ so that Eq. (4) reduces to

$$\Phi(x) = F(x) + \int_0^x \Phi(\eta)K(x-\eta)\, d\eta + \int_x^L \Phi(\eta)K(\eta-x)\, d\eta \tag{14}$$

This equation will now be solved by the same methods used previously.

Approximate separable kernel. - Since the kernel K is being approximated by e^{-2z}, we make a similar approximation for the configuration factor $F(x)$. The quantity $2F(x)$ is plotted on Fig. 4 and it is seen to be quite close to an exponential function so we let $2F(x) = e^{-\beta x}$. To determine β we impose the condition that the total radiant exchange by the exact and approximate form factors would be the same for a cylinder of infinite length. This gives

$$2\int_0^\infty \left[\frac{x^2 + 1/2}{(x^2+1)^{1/2}} - x\right] dx = \int_0^\infty e^{-\beta x}\, dx$$

which yields $\beta = 2$. Eq. (14) then becomes

$$\Phi(x) = \frac{1}{2}e^{-2x} + \frac{1}{e^{2x}}\int_0^x \Phi(\eta)e^{2\eta}\, d\eta + e^{2x}\int_x^L \Phi(\eta)\,\frac{1}{e^{2\eta}}\, d\eta \tag{15}$$

By differentiating this integral equation twice, the integrals are eliminated and a simple differential equation is obtained

$$\frac{d^2\Phi}{dx^2} = 0 \tag{15a}$$

This yields the straight line solution

$$\Phi = C_3 x + C_4 \tag{16}$$

By evaluating Eq. (15) at $x = 0$ and $x = L$ the two boundary conditions for this solution are found to be

$$\Phi(0) = \frac{1}{2} + \int_0^L \Phi(\eta)\, \frac{1}{e^{2\eta}}\, d\eta$$

$$\Phi(L) = \frac{1}{2}\, e^{-2L} + e^{-2L} \int_0^L \Phi(\eta) e^{2\eta}\, d\eta \tag{17}$$

Substituting Eq. (16) into these relations and evaluating the integrals provides two equations from which the constants C_3 and C_4 can be determined. This yields the final result

$$\Phi = \frac{1}{1 + L} \left(\frac{1}{2} + L - x \right) \tag{18}$$

Numerical integration. - To examine the accuracy of the approximate result in Eq. (18), numerical computations were performed for the same two enclosures, $L = 1$ and 4, considered previously for the case with uniform heat flux at the enclosure walls. As seen from the table the results are in very good agreement with the simple expression given in Eq. (18). To obtain numerical values for $x > L/2$ note that the Φ function is asymmetric about $L/2$ so that $\Phi(L - x) = 1 - \Phi(x)$.

Variational method. - To find the best straight line relation for Φ we again employ the variational method. The variational principle for this case is

$$J = \int_0^L \int_0^L K(|\eta - x|)\Phi(\eta)\Phi(x)\, dx\, d\eta - \int_0^L [\Phi(x)]^2\, dx + 2 \int_0^L \Phi(x) F(x)\, dx \tag{19}$$

We then let $\Phi = D + Ex$ and carry out the indicated operations. Most of the necessary algebra has already been performed in relation to Eqs. (10) and (11). This yields J as a function of the constants D and E, and we now obtain an extremum by differentiating J separately with respect to D and E and setting the resulting equations equal to zero. This yields,

$$\frac{\partial J}{\partial D} = 0 \longrightarrow D(-L^2 + L\sqrt{L^2+1}) + \frac{E}{2}(-L^3 + L^2\sqrt{L^2+1}) + \frac{L^2}{2} - \frac{L}{2}\sqrt{L^2+1} = 0 \tag{20a}$$

$$\frac{\partial J}{\partial E} = 0 \longrightarrow D(-L^2 + L\sqrt{L^2+1}) + \frac{E}{2}\left[-L^3 + \left(\frac{1}{2} + L^2\right)\sqrt{L^2+1} - \frac{1}{2L}\ln\left(L + \sqrt{L^2+1}\right)\right] + \frac{(2L^2-1)\sqrt{L^2+1} + 1 - 2L^3}{3L} = 0 \tag{20b}$$

Solving simultaneously we obtain

$$D(L) = \frac{-4L^5 + 5L^3 + 4L^2 + 7L + (4L^4 - 7L^2 - 4L)\sqrt{L^2+1} + 3(L - \sqrt{L^2+1})\ln(L + \sqrt{L^2+1})}{6[L^3 + L - L^2\sqrt{L^2+1} + (L - \sqrt{L^2+1})\ln(L + \sqrt{L^2+1})]}$$

and

$$E(L) = \frac{2}{3}\left[\frac{-L^3 + 2 + (L^2 - 2)\sqrt{L^2+1}}{L\sqrt{L^2+1} - \ln(L + \sqrt{L^2+1})}\right] \tag{21}$$

The coefficients D and E are plotted for a range of L values on Fig. 5 and are compared with the more simple coefficients resulting from the separable kernel method. The values of Φ for L = 1 and 4 are shown in the table and agree quite well with the previous solutions.

NONBLACK SURFACE

We shall now consider surfaces which are gray and are diffuse reflectors. For the cases considered in this paper, where the heat flux is specified and the wall temperature is found, a simple relation will be determined relating the solution for the black body problem to the diffuse gray body case. Consider a function U(x) which is defined as the total amount of energy leaving a surface element by radiation. If we form a heat balance on an element of area, the total energy leaving is equal to the imposed heat flux plus the radiation coming from the ends of the enclosure and the enclosure walls. This gives

$$U(x) = q(x) + \varphi_1 F(x) + \varphi_2 F(L-x) + \int_0^x U(\eta)K(x-\eta)\,d\eta + \int_x^L U(\eta)K(\eta-x)\,d\eta \tag{22}$$

This equation is of exactly the same form as Eq. (4) and hence U(x) is equal to the solution for the emissive power previously obtained for the case of black body surfaces.

Now that $U(x)$ is known we have to relate it to the surface temperature. To do this consider in detail the quantities which contribute to the energy leaving the surface. One term is the energy emitted directly from the surface. A second term is the energy arriving from the ends of the enclosure which is reflected by the surface, and the final term is the energy arriving from the other portions of the enclosure which is reflected. Thus $U(x)$ can be written as

$$U(x) = \epsilon(x)\varphi_w(x) + \rho(x)\left[\varphi_1 F(x) + \varphi_2 F(L-x) + \int_0^x U(\eta)K(x-\eta)\,d\eta + \int_x^L U(\eta)K(\eta-x)\,d\eta\right] \tag{23}$$

If we multiply Eq. (22) by $\rho(x)$ and subtract Eq. (23) we can then obtain the solution for the wall emissive power

$$\varphi_w(x) = \frac{\rho}{\epsilon}\,q + U \tag{24}$$

where, for constant q, we have by using the black body solution Eq. (5)

$$U(x) = \varphi_2 + q\Phi_q^*(x) + (\varphi_1 - \varphi_2)\Phi(x)$$

If the surface is made a better reflector ($\epsilon \to 0$ and $\rho \to 1$) the φ_w distribution becomes higher.

SUMMARY

For the case considered here where the wall heat input is specified, it is shown in the analysis that solving the energy equation for black body surfaces is equivalent to solving an energy equation for gray body surfaces. A simple expression was found that relates the two solutions. This relation between black body temperature distributions and gray body temperature distributions is true for any enclosure provided both the geometry and heat fluxes are identical. For clearness of presentation black body surfaces were considered first and then the extension was made to the nonblack case.

Although the integral equation expressing the energy balance is linear, the complicated algebraic form of the kernel presents analytical difficulties. These can be eliminated by approximating the kernel by a simple separable kernel which meets certain conservation requirements. Solutions can then be found for the energy

equation which check fairly well with numerical calculations. These approximate solutions are of a simple functional form. It is then logical to ask of the solution could be approximated more closely by functions of the same form but with optimum coefficients. By applying the Ritz method to a variational principle for the energy equation, these optimum coefficients can be obtained and the solution is in very close agreement with numerical solutions calculated for specific values of the parameter L. It is possible that this method of attack may be of value in other radiation problems involving complicated kernels.

APPENDIX

HEAT INPUT FROM WALLS OF CYLINDRICAL ENCLOSURE

The heat input from an element dA_2 to dA_1, as shown in Fig. 2, is

$$dq_{21} = \frac{\sigma T_w^4(\eta')}{\pi} \cos\theta_2 \, d\omega_2 \, dA_2 \tag{25}$$

where ω_2 is the solid angle of dA_1 seen by dA_2. Using simple geometrical relations the following relations can be obtained,

$$d\omega_2 = \frac{\cos\theta_1 \, dA_1}{d^2} \qquad d^2 = 2R^2(1 - \cos\theta) + (\eta' - x')^2$$

$$\cos\theta_1 = \cos\theta_2 = \frac{R(1 - \cos\theta)}{d} \qquad dA_2 = Rd\eta' \, d\theta$$

Substituting into Eq. (25) we have

$$dq_{21} = \frac{\sigma T_w^4(\eta')}{\pi} \frac{R^3(1 - \cos\theta)^2}{\left[2R^2(1 - \cos\theta) + (\eta' - x')^2\right]^2} d\theta \, d\eta' \, dA_1$$

Since the temperature is only a function of the axial dimension an integration in θ can be performed. Thus the heat input from an elemental ring at η' to dA_1 is

$$dq_{21} = \frac{\sigma T_w^4(\eta')}{2\pi} \int_0^{2\pi} \frac{(1 - \cos\theta)^2}{\left[1 - \cos\theta + 2\left(\frac{\eta' - x'}{2R}\right)^2\right]^2} d\theta \, d\left(\frac{\eta'}{2R}\right) dA_1 \tag{26}$$

This integral was evaluated by methods of complex variables. Using x and η as nondimensional lengths the heat flux from the elemental ring to dA_1 is

$$\frac{dq_{21}}{dA_1} = \sigma T^4(\eta) \left\{ 1 - \frac{[\eta - x]^3 + \frac{3}{2}[\eta - x]}{[1 - (\eta - x)^2]^{3/2}} \right\} d\eta$$

Thus the total heat input per unit area from all areas of the enclosure to the right of the element of area at x is

$$\frac{q_{21}}{dA_1} = \int_x^L \sigma T^4(\eta) \left\{ 1 - \frac{[\eta-x]^3 + \frac{3}{2}[\eta-x]}{[1 + (\eta-x)^2]^{3/2}} \right\} d\eta \qquad (27)$$

The result for the contribution from the portion of the enclosure to the left of x is of the same form except that the quantity $\eta-x$ is replaced by $x-\eta$.

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TABLE I. - DISTRIBUTIONS OF Φ_q^* AND Φ

		Φ_q^*			Φ		
	x	Numerical	Separable kernel	Variational	Numerical	Separable kernel	Variational
L=1	0	2.06	2.00	2.06	0.760	0.750	0.759
	1/8	2.31	2.19	2.31	.697	.688	.696
	1/4	2.49	2.37	2.49	.632	.625	.630
	3/8	2.60	2.47	2.60	.567	.563	.565
	1/2	2.64	2.50	2.63	.501	.500	.500
L=4	0	5.1*	5.00	4.98	0.905*	0.900	0.896
	1/2	8.7	8.50	8.60	.805	.800	.797
	1	11.4	11.0	11.2	.705	.700	.698
	3/2	12.8	12.5	12.7	.604	.600	.599
	2	13.3	13.0	13.2	.502	.500	.500

* Will be supplied to greater accuracy

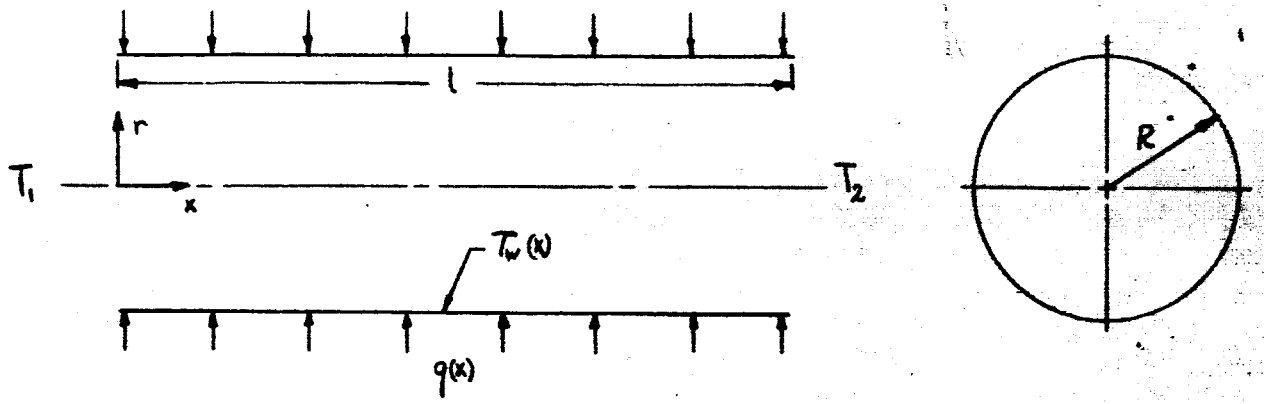


Fig. 1 Hollow cylinder enclosure

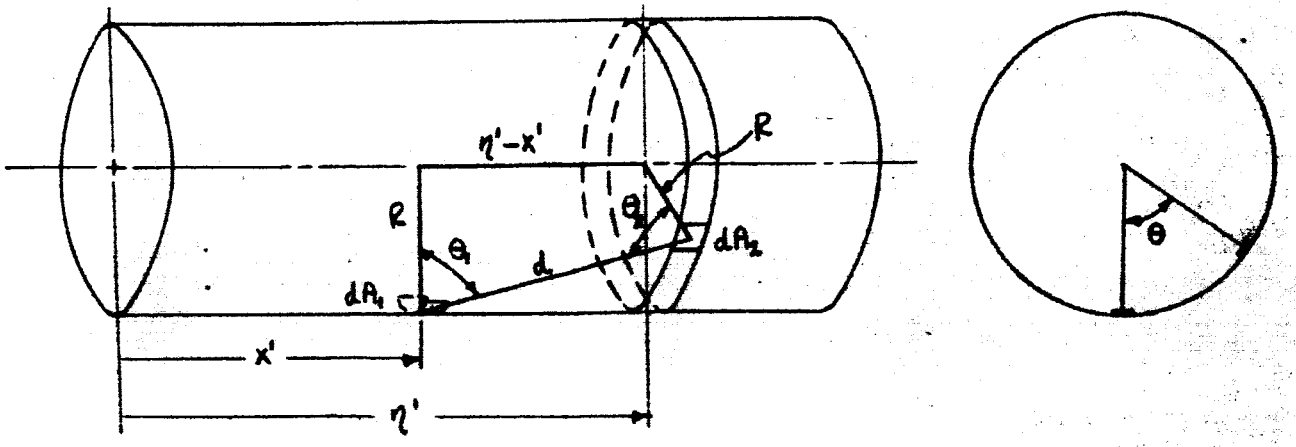


Fig. 2 Coordinates for determining configuration factor

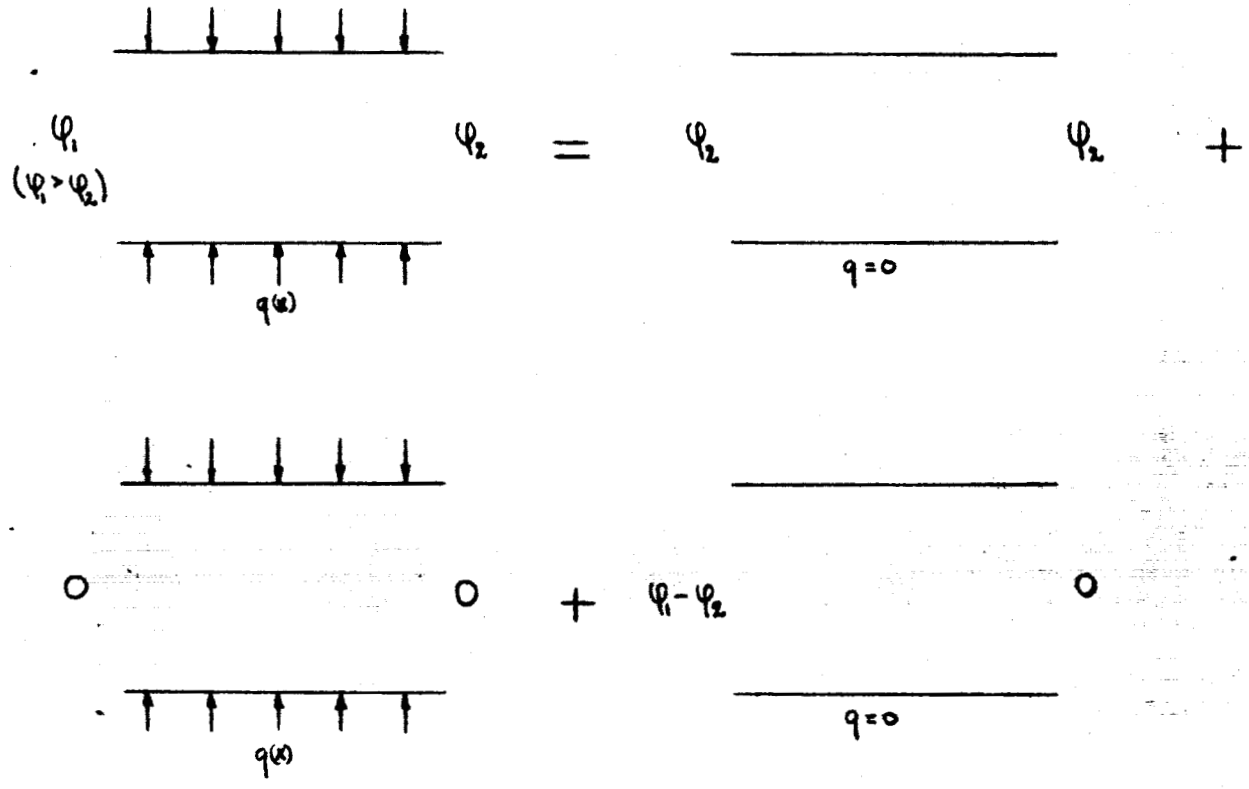


Fig.3 Superposition of elementary solutions

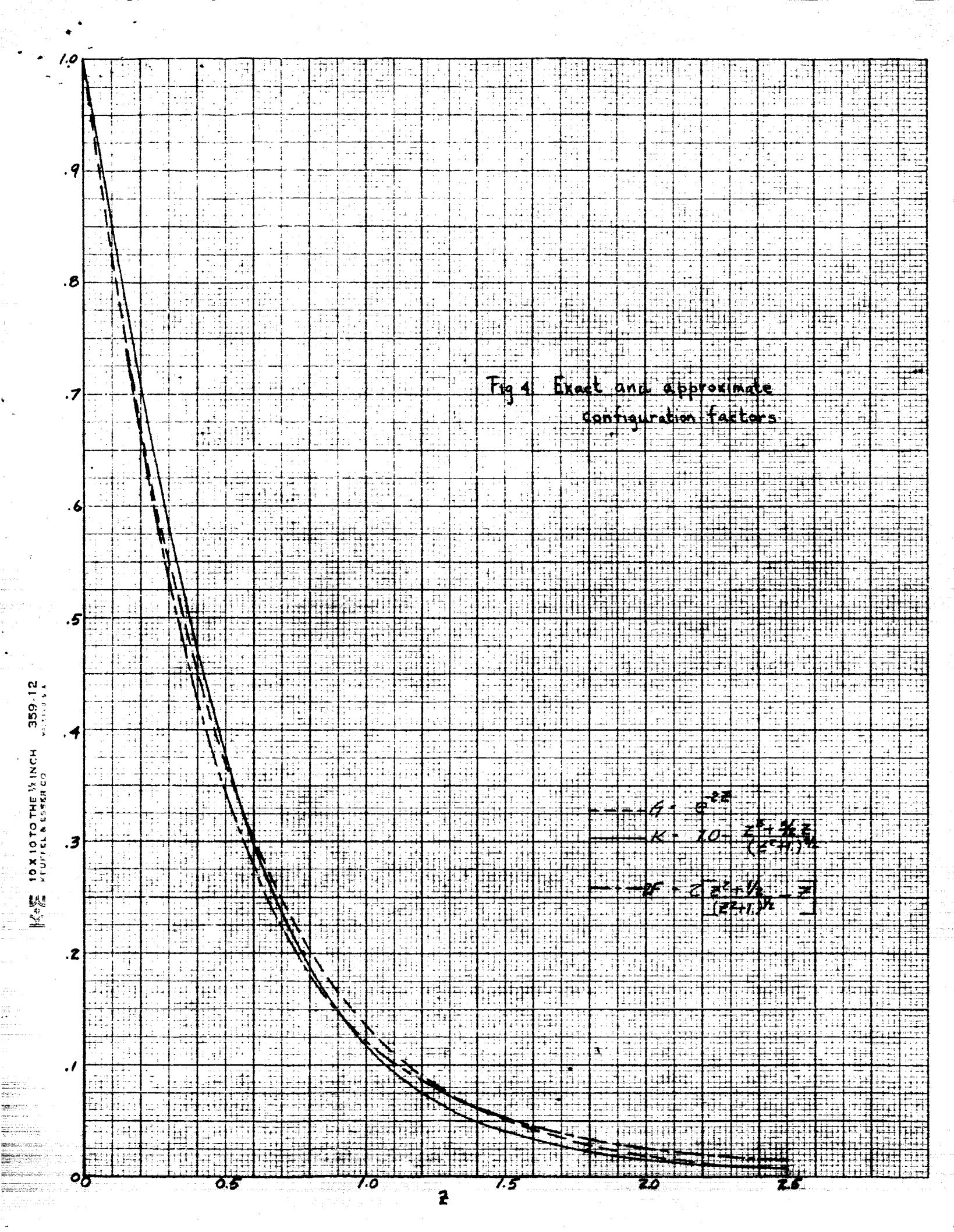
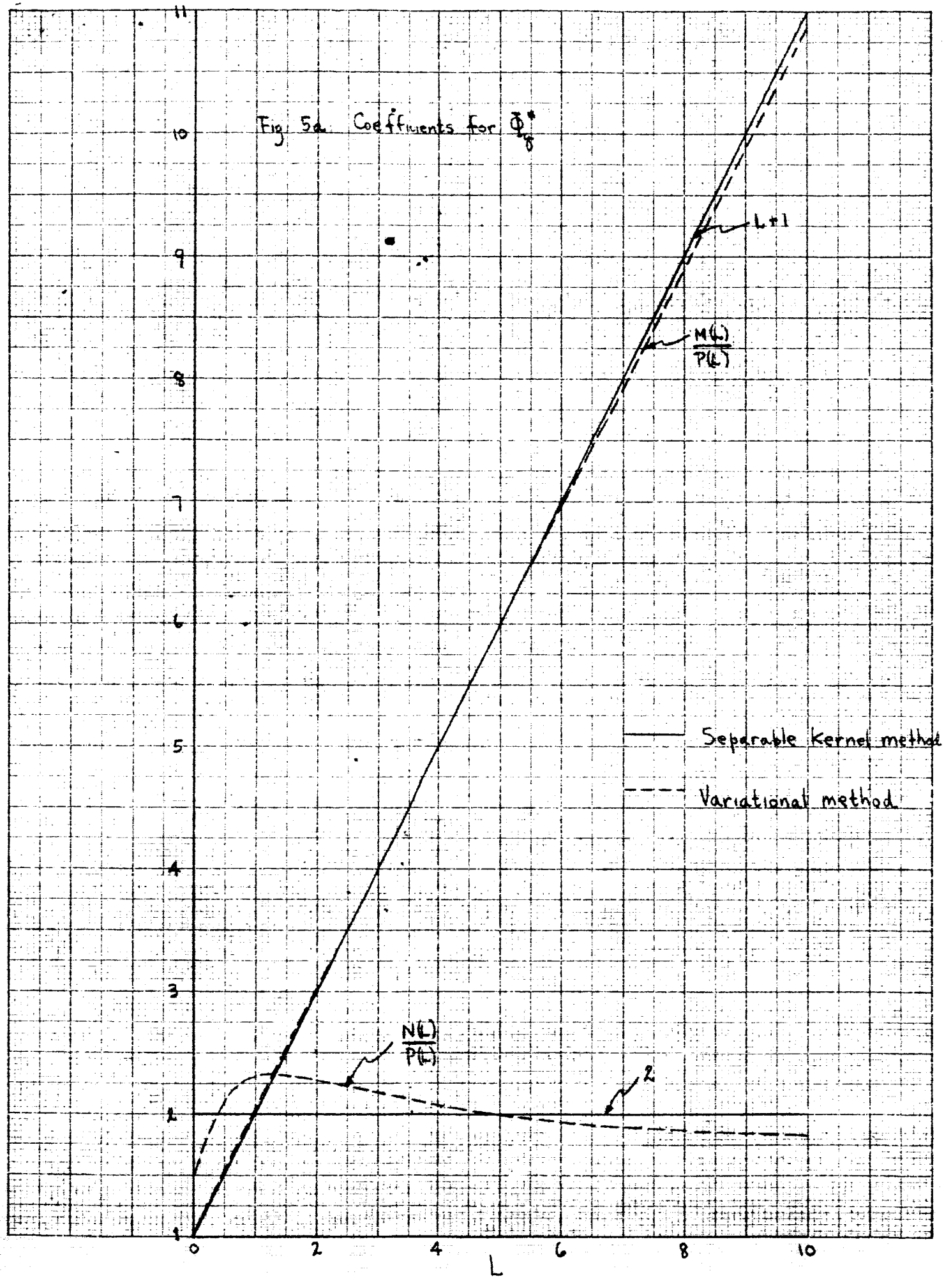


Fig 4 Exact and approximate configuration factors

10 X 10 TO THE ½ INCH 359-12
KEUFFEL & ESSER CO



359-12
10 X 10 TO THE ½ INCH
KEUFFEL & ESSER CO.

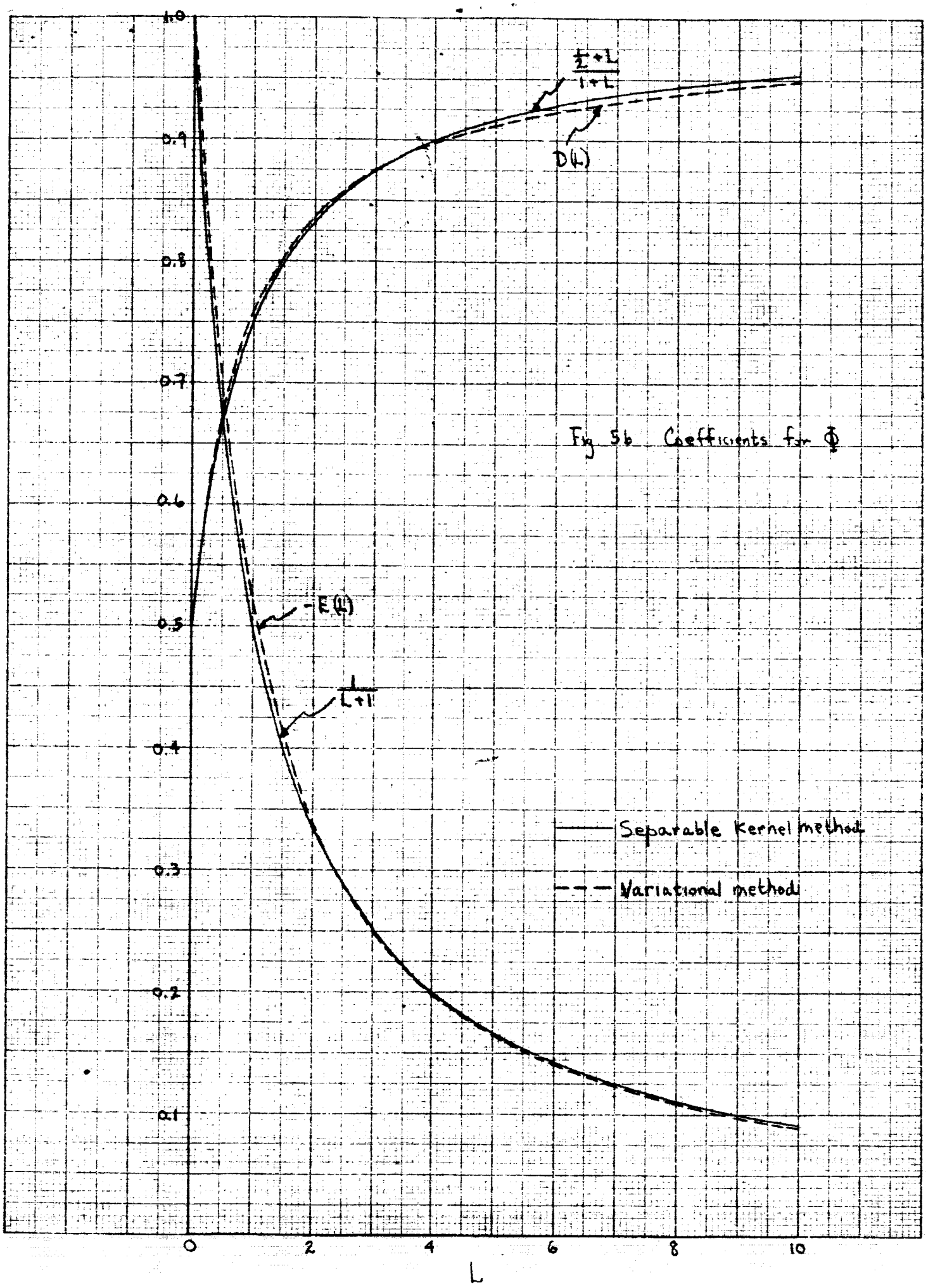


Fig 5b Coefficients for Φ

359.12
10 X 10 TO THE INCH
KEUFFEL & ESSER CO.